ROLL NO:								

Candidate must write code on the title page of answer book

1. Please check this question paper contains 9 printed pages
2. Code number given in the right hand side of the question paper should be written on the title page of the answer book by the candidate.
3. Please check that this question paper contains 33 of questions
4. Please write down the serial number of question papers before attempting it
5. Fifteen minutes are allotted to read this question paper during this time student will

Time Allowed: 3.00Hrs. Maximum Marks: 70

General Instructions:

1. This question paper contains two **Parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks.
2. **Part A** has Objective Type Questions and **Part B** has Descriptive Type Questions.
3. Both Part A and Part B have choices.

Part A:

1. It consists of two sections - **I and II**.
2. Section I comprises of 16 very short answer type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.
4. Internal choice is provided in **5** questions of Section - I. Moreover internal choices have been given in both questions of Section - II as well.

Part B:

1. It consists of three sections - **III, IV and V**.
2. Section III comprises of 10 questions of **2 marks** each.
3. Section IV comprises of 7 questions of **3 marks** each.
4. Section V comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of Section - III, **2** questions of Section - IV and **3** questions of Section - V. You have to attempt only one of the alternatives in all such questions.

PART - A

Section I

Questions in this section carry 1 mark each.

Q01. Let $(a_1, a_2) \in R$ for all $a_1, a_2 \in A$, such that the relation R is defined in a set A. What should be added to the relation R for making it a symmetric relation?

OR

In a relation R on A, if each element of A is related to itself only, then name the relation R.

Q02. State if the relation $R = \{(1,2),(2,3),(1,3)\}$, which is defined on $A = \{1,2,3\}$, is transitive or not. Justify your answer.

Q03. Let A and B be two sets. State if $f: A\times B \to B\times A$ such that $f(a,b)=(b,a)$ is onto function or, not. Justify your answer.

OR

What is the range of the function $f(x)=\dfrac{(x-1)}{|x-1|}$, $x\neq 1$?

Q04. If A is a square matrix such that $A^2 = A$, then write the value of $(I-A)^3 + A$.

Q05. If A is a matrix of order 3×2, then the find the order of the matrix A'.

OR

Let $A=\begin{bmatrix} 4 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find $(adj.A)A$.

Q06. For what value of k, $\begin{bmatrix} 2 & -1 & 3 \\ k & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not an invertible matrix?

Q07. Write the value of $\int \dfrac{e^x dx}{9+e^{2x}}$.

OR

Evaluate $\int \dfrac{3^{5\log_3 x} + 3^{6\log_3 x}}{3^{4\log_3 x} + 3^{5\log_3 x}} dx$.

Q08. Find the area bounded by $y = 4 - x^2$ with x-axis. Use integrals.

Q09. Write the primitive of $x^{2019x}(1+\log x)$.

OR

Determine the value of $\int_{-\pi/4}^{\pi/4} \left(\cos^{-1} x + \sin^{-1} x\right) dx$.

Q10. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then find the value of λ.

Q11. The position vectors of two points A and B are $\overrightarrow{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\overrightarrow{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. Find the position vector of a point P which bisects the line segment joining the points A and B.

Q12. Find the slope of the tangent to the curve $y = 4^x$ at (0, 2).

Q13. Find a unit vector parallel to $2\vec{a} - \vec{b} + 3\vec{c}$, where

$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$,

$\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, and

$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$.

Q14. For what value (s) of x, the function $f(x) = x^2 - 2x$ is an increasing function?

Q15. A speaks truth in 80% cases and B speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact?

Q16. An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Write the probability that they are of the different colours.

SECTION II

Questions in this section carry 1 mark each.

Both the Case study based questions are compulsory. **Attempt any 4 sub-parts from each question 17 (i-v) and 18 (i-v)**.

Q17. A butterfly is moving in a straight path in the space.

Let this path be denoted by a line l whose equation is $\frac{x-1}{2}=\frac{2-y}{3}=\frac{z-3}{4}$ *say*.

Using the information given above, answer the following with reference to the line l :

(i) The position vector of the point on the line is

(a) $\hat{i}+2\hat{j}+3\hat{k}$

(b) $\hat{i}+2\hat{j}+\hat{k}$

(c) $2\hat{i}+3\hat{j}+4\hat{k}$

(d) $2\hat{i}-3\hat{j}+4\hat{k}$

(ii) What are the direction ratios of the line?

(a) 2, 3, 4

(b) –2, 3, 4

(c) 2, –3, 4

(d) 2, 3, –4

(iii) If the z-coordinate of a point on this line is 11, then the x-coordinate of the same point on this line, is

(a) –5

(b) 5

(c) 0

(d) 1

(iv) The vector equation of the given line is

(a) $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(2\hat{i}-3\hat{j}+4\hat{k})$

(b) $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$

(c) $\vec{r}=\hat{i}-2\hat{j}+3\hat{k}+\lambda(2\hat{i}-3\hat{j}+4\hat{k})$

(d) $\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \lambda(2\hat{i} - 3\hat{j} + 4\hat{k})$

(v) The unit vector in the direction of the vector parallel to the given line, is

(a) $\dfrac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$

(b) $\dfrac{2\hat{i} + 3\hat{j} + 4\hat{k}}{\sqrt{29}}$

(c) $\dfrac{\hat{i} - 2\hat{j} + 3\hat{k}}{\sqrt{14}}$

(d) $\dfrac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$

Q18. The reliability of a HIV test is specified as follows:
Of people having HIV, 90% of the test detect the disease but 10% go undetected.
Of people free of HIV, 99% of the test are judged HIV negative but 1% are diagnosed as showing HIV positive.

From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV positive.

Based on the above information, answer the following :

(i) What is the probability of the person to be tested as HIV positive given that he is actually having HIV?

(a) 0.001

(b) 0.1

(c) 0.8

(d) 0.9

(ii) What is the probability of the,person to be tested as HIV positive given that The is actually not having HIV?
(a) 0.01
(b) 0.99
(c) 0.1
(d) 0.001

(iii) What is the probability that the person is actually not having HIV?
(a) 0.998
(b) 0.999
(c) 0.001
(d) 0.111

(iv) What is the probability that the person is actually having HIV given that the is tested as HIV positive?
(a) 0.83
(b) 0.0803
(c) 0.083
(d) 0.089

(v) What is the probability that the ‘person selected will be diagnosed as HIV positive?
(a) 0.1089
(b) 0.01089
(c) 0.0189
(d) 0.189

PART - B

Section III

Questions in this section carry 2 marks each.

Q19. Simplify : $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right),\ 0 < x < \dfrac{\pi}{2}$.

Q20. For $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, find inverse of A. Then, show that $A^{-1}A = I$.

OR

If $\cos 2x = 0$, then find the numeric value of $\begin{vmatrix} 0 & \cos x & \sin x \\ \cos x & \sin x & 0 \\ \sin x & 0 & \cos x \end{vmatrix}^2$.

Q21. Differentiate $\log[\log(\log x^5)]$ w. r. t. x.

Q22. Using derivatives, find the minimum value of $ax + by,$ where $xy = r^2,\ a > 0,\ b > 0$.

Q23. Find $\displaystyle\int_{\pi}^{2\pi} \frac{x + \cos 6x}{3x^2 + \sin 6x}\,dx$.

OR

Find $\displaystyle\int \left(\frac{1-x}{1+x^2}\right)^2 e^x\,dx$.

Q24. Find the area bounded by $4x = y^2$ with its latus-rectum.

Q25. Solve the differential equation $\dfrac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

Q26. Find the Cartesian and vector equations of the plane which passes through the point (–2, 4, –5) and perpendicular to the line given by $10(x+3) = 6(y-4) = 5(z-8)$.

Q27. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

Q28. Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

OR

The probability of simultaneous occurrence of at least one of two events A and B is p. If the probability that exactly one of A, B occurs is q, then prove that $P(A') + P(B') = 2 - 2p + q$.

Section IV

Questions in this section carry 3 marks each.

Q29. Show that the function $f : (-\infty, 0) \to (-1, 0)$ defined by $f(x) = \dfrac{x}{1+|x|},\ x \in (-\infty, 0)$ is one-one and onto.

Q30. If $y = \dfrac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left(\sqrt{\dfrac{a-b}{a+b}} \tan\dfrac{x}{2}\right)$, show that $\dfrac{d^2y}{dx^2} = \dfrac{b\sin x}{(a + b\cos x)^2}$.

OR

If $\dfrac{x}{x-y} = \log\dfrac{a}{x-y}$, then prove that $\dfrac{dy}{dx} = 2 - \dfrac{x}{y}$.

Q31. Discuss the continuity of $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, \text{ if } x\neq 0 \\ 0, \text{ if } x=0 \end{cases}$ at $x=0$.

Is it differentiable at the same point?

Q32. Find the coordinates of a point P *say*, where the line passing through the points A (3, 4, 1) and B (5, 1, 6) crosses the plane whose intercepts on the x, y and z-axes are respectively $\frac{7}{2}$, 7, 7 .

Q33. Integrate $\dfrac{\sin x+\cos x}{\sin^2 x+\cos^4 x}$ w. r. t. x.

Q34. Find the points of intersection of the ellipse $x^2+9y^2=10$ and the line $x=1$.

Hence using integration, find the area bounded between the given ellipse and line.

Q35. Solve the differential equation: $\operatorname{cosec} x \log y \; dy + x^2 y^2 \; dx = 0$.

OR

Find the particular solution of $(x-y)(dx+dy)=dx-dy$, where $y(0)=-1$.

Section V

Questions in this section carry 5 marks each.

Q36. Solve the following system of equations by matrix method:

$x-y+2z=7$,

$2x-y+3z=12$,

$3x+2y-z=5$.

OR

If A and B are square matrices of the same order such that AB = BA, then prove by using induction that $AB^n=B^nA$. Further, prove that $(AB)^n=A^nB^n$ for all $n\in N$.

Q37. If $x\cos\alpha+y\sin\alpha=p$ touches the curve $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$, then prove that $a^2\cos^2\alpha-b^2\sin^2\alpha=p^2$.

OR

Find the intervals in which the following function is strictly increasing or strictly decreasing. Also find the points of local maximum and local minimum, if any:

$f(x)=(x-1)^3(x+2)^2$.

Q38. Use graphical method to maximize: $Z=(100x+120y)$

Subject to constraints:

$x\geq 0$,

$y\geq 0$,

$5x+8y\leq 200$,

$10x+8y\leq 240$.

OR

Solve the following linear programming graphically.

To minimize: $Z=6x+3y$

Subject to the constraints :

$12x+3y\geq 240,\ 4x+20y\geq 460,\ 6x+4y\leq 300,\ x\geq 0,\ y\geq 0$.

Also, write the values of x and y at which the minimum value of Z is obtained.